BID: Thermal expansion

The Big Idea

When they get hot, most substances expand. When they get cooler, most substances shrink.

More details

Over certain temperature ranges for a wide variety of substances, there is a percent change in the length or volume and it's proportional to the change in temperature. Each substance has its own proportionality constant for this change called the thermal expansion coefficient.

The formula is given as:

$\Delta\ell = \alpha\, \ell_o\, \Delta T$ ($\Delta\ell$ = change in length, α = linear coefficient, ℓ_o = orig. length, ΔT = temp change)

I prefer the form $\Delta\ell/\ell_o = \alpha\, \Delta T$ which essentially states that the percent change in length is equal to a constant for a given material times how many degrees the temperature changes.

The coefficient has the units $°C^{-1}$ or "per degree C" and describes the change in length of the substance per degree change in temperature (it doesn't matter if you use kelvins or °C as they are both the same size). The units for length don't matter as long as the length and change in length are measured in the same units.

For volume expansion, the formula is basically the same format, just a different coefficient: $\Delta V = \beta\, V_o\, \Delta T$

How to recognize it

The problem will most often give you a coefficient of thermal expansion and then either a change in temp or a change in size and ask that you find the other. Occasionally, the problem may give you the temps and sizes and ask you to find the coefficient instead.

How to tackle it

Just use the formula to plug and chug for the unknown quantity

Pitfalls to watch for

1) Students often are asked for the final length and give the change in length or vice versa.
2) Same for temperatures: be sure to double check if they're asking for final temp or change in temp, and do the appropriate math.
3) If the problem has two adjacent objects (eg railroad rails, sidewalk sections, etc.), then the solution for how big a gap is or how much it can expand depends on BOTH sides expanding or contracting. Be very careful of this! A diagram would help avoid mistakes here.
4) If the final temp is lower than initial temp, then the change in temp will be negative, and the expansion should be negative as well.

Example problem

A copper pipe is measured to be 8.0 m long on a cold day: - 10 °C. How long would this same pipe measure on a day where T = 40 °C? ($\alpha = 16.6 \times 10^{-6}$ per °C)

Solution:

$\Delta\ell = \alpha\, \ell_o\, \Delta T$

$\Delta\ell = (16.6 \times 10^{-6}\ °C^{-1}) \cdot 8.0\ m \cdot 50\ °C = 0.00664\ m$ or 6.64 mm.

$\ell_f = \ell_o + \Delta\ell = 8.0\ m + 0.00664\ m = \boxed{8.00664\ m}$